

# **Learning dynamics for robot control under varying contexts**

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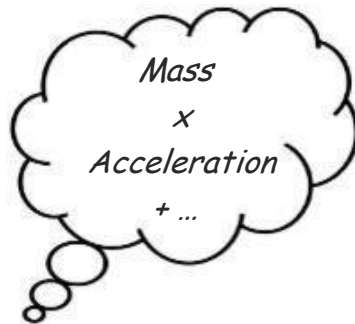
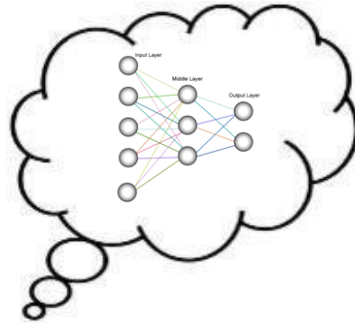
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# Motor control



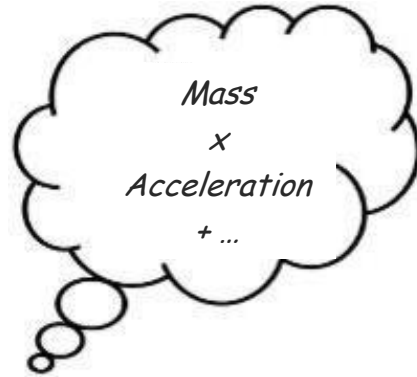
- Biological motor systems show a remarkable level of adaptability and robustness under different conditions
- We are interested to achieve similar levels of ability in robots as well

# Motor control / dynamics



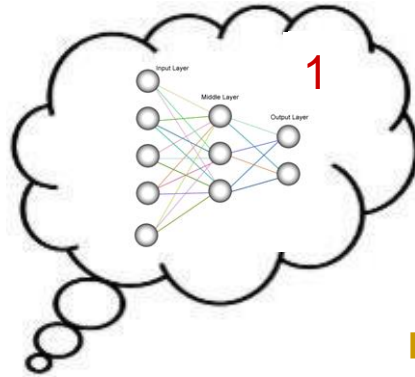
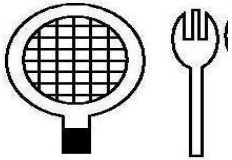
- High performance control requires an **accurate dynamics model**
- Usually model is derived using knowledge of the physical properties of the robot

# Learning dynamics



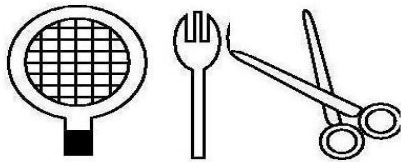
- Accurate derivation of dynamics may not be possible due to:
  - Unmodeled effects
  - Inaccurate knowledge or complete lack of knowledge of physical parameters
- Alternative is to learn the dynamics

# Nonstationarity

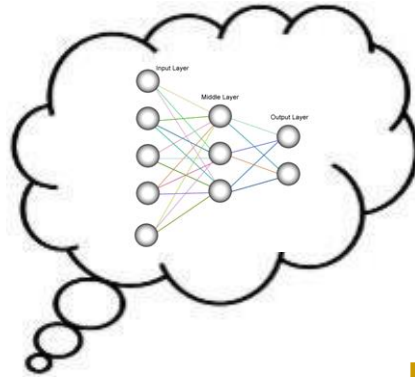


- Dynamics may change due to **interaction** with different environments and objects. Varying **context** of the dynamics.
- Model needs to be adapted every time the context changes
- The context is in most cases not directly observed

# Goals



1 ? >1

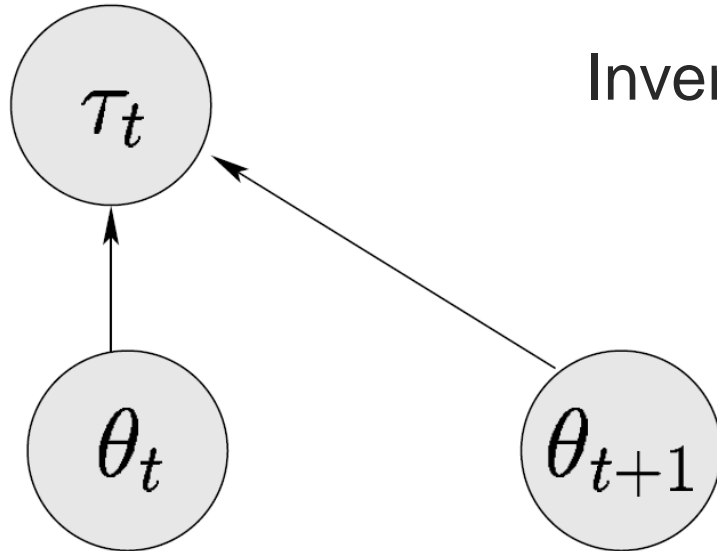


- However, if dynamics **switch back and forth**, readapting every time is suboptimal. Need to cope with **recurring contexts**.
- What happens with novel contexts? We would like to **generalize from seen contexts to novel contexts**.

# Overview

1. Learning **single context** dynamics for control
  2. Learning multiple models for a set of **recurring contexts**
  3. Learning a single model with continuous latent contextual variables for a **range of varying contexts**
  4. Learning a single **model with observed continuous contextual variables**
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# Learning single context dynamics



Inverse dynamics for a single context

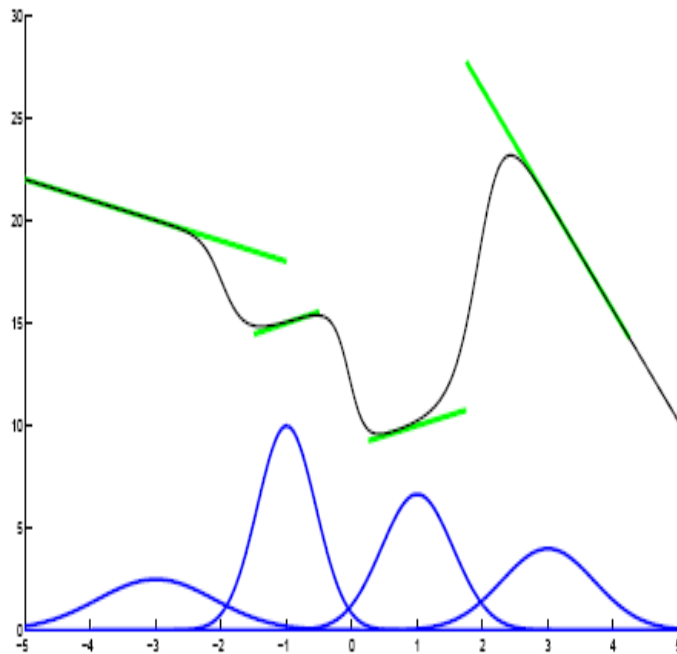
$$\tau_t = g(\Theta_t, \Theta_{t+1})$$

Learning the inverse model for a single context is (relatively) straight forward since all data is observed, we just need a learning algorithm

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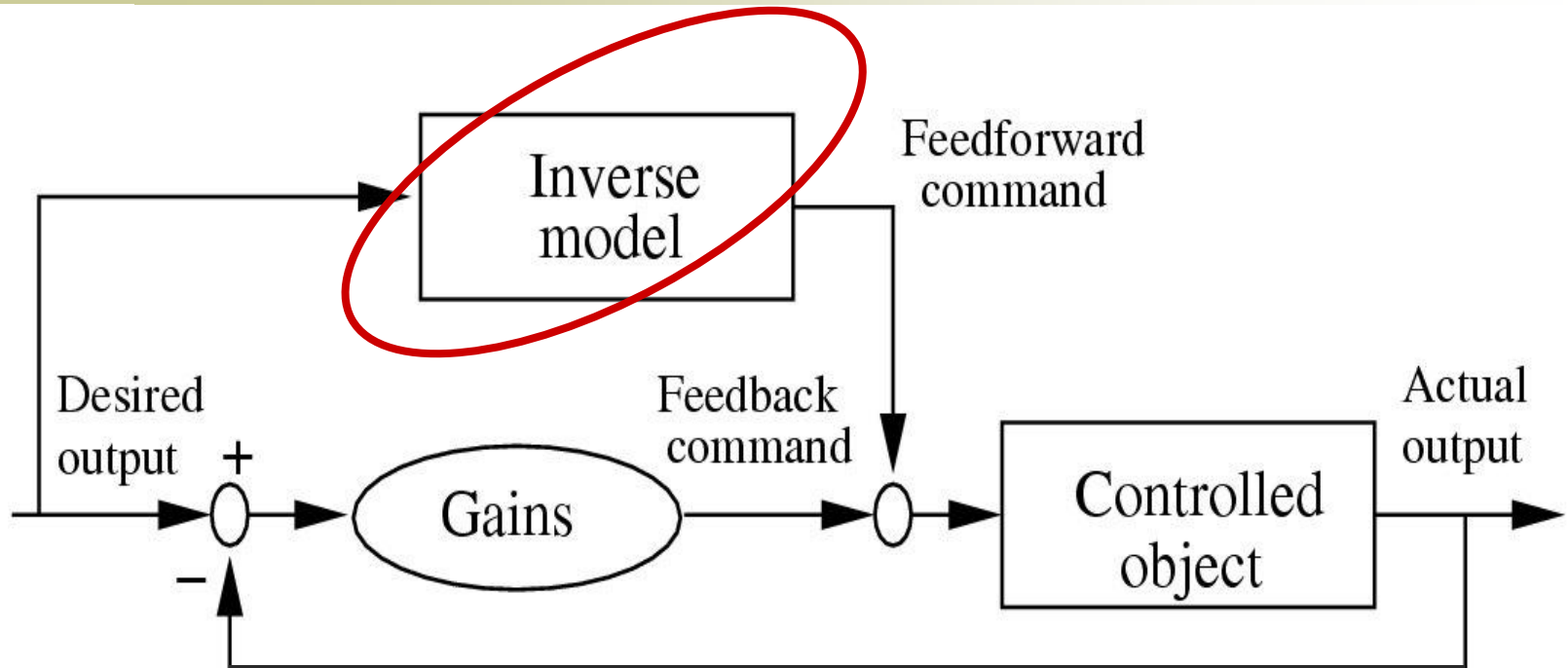
# Locally Weighted Projection Regression



- Pairs of linear models and gaussian locality kernels
- Overall prediction weighted sum of individual models' predictions
- Local model PLS
- Online learning of local projection directions, regression coefficients and kernel shape
- Provides input dependent confidence bounds
- Appropriate for motor learning tasks in high dimensional spaces

Vijayakumar, D'Souza and Schaal. Incremental online learning in high dimensions. Neural Computation, 17:2602-2634

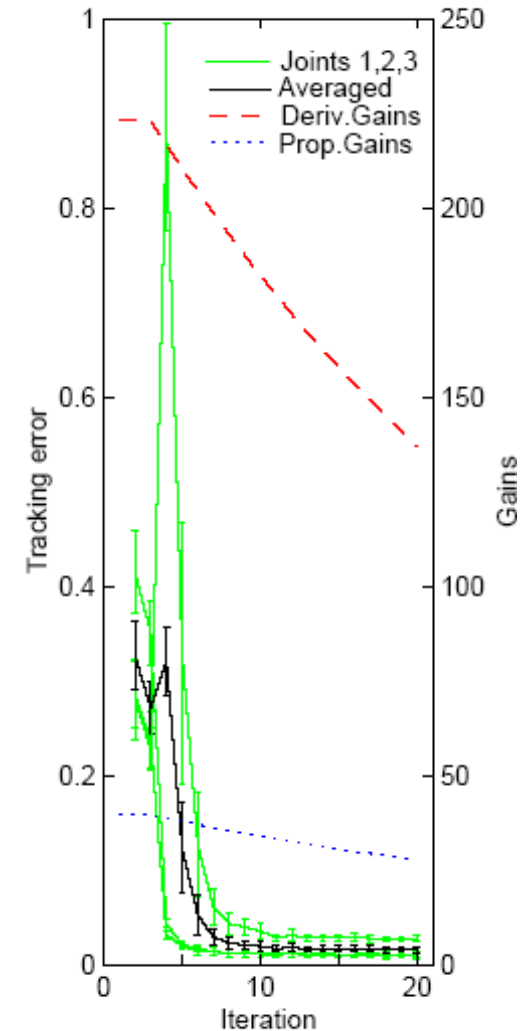
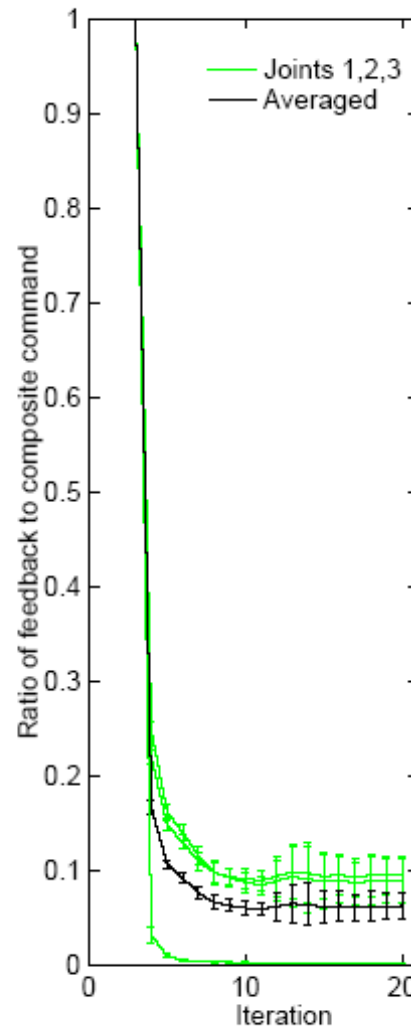
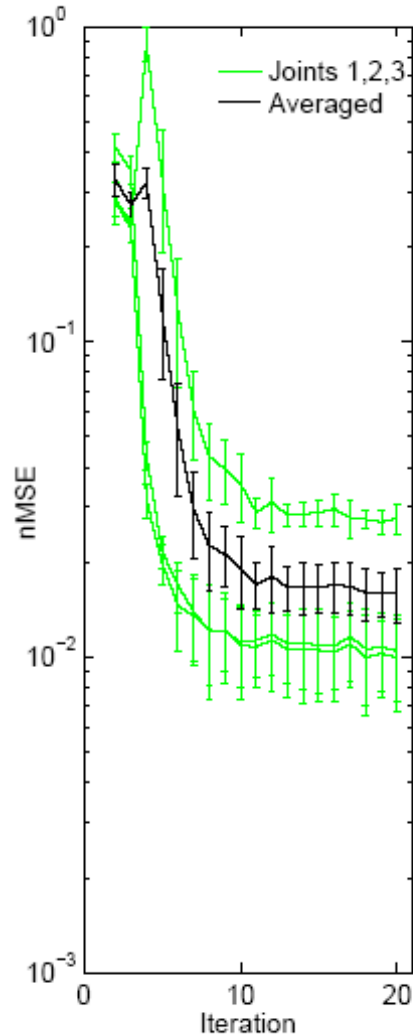
# ...and using it for control



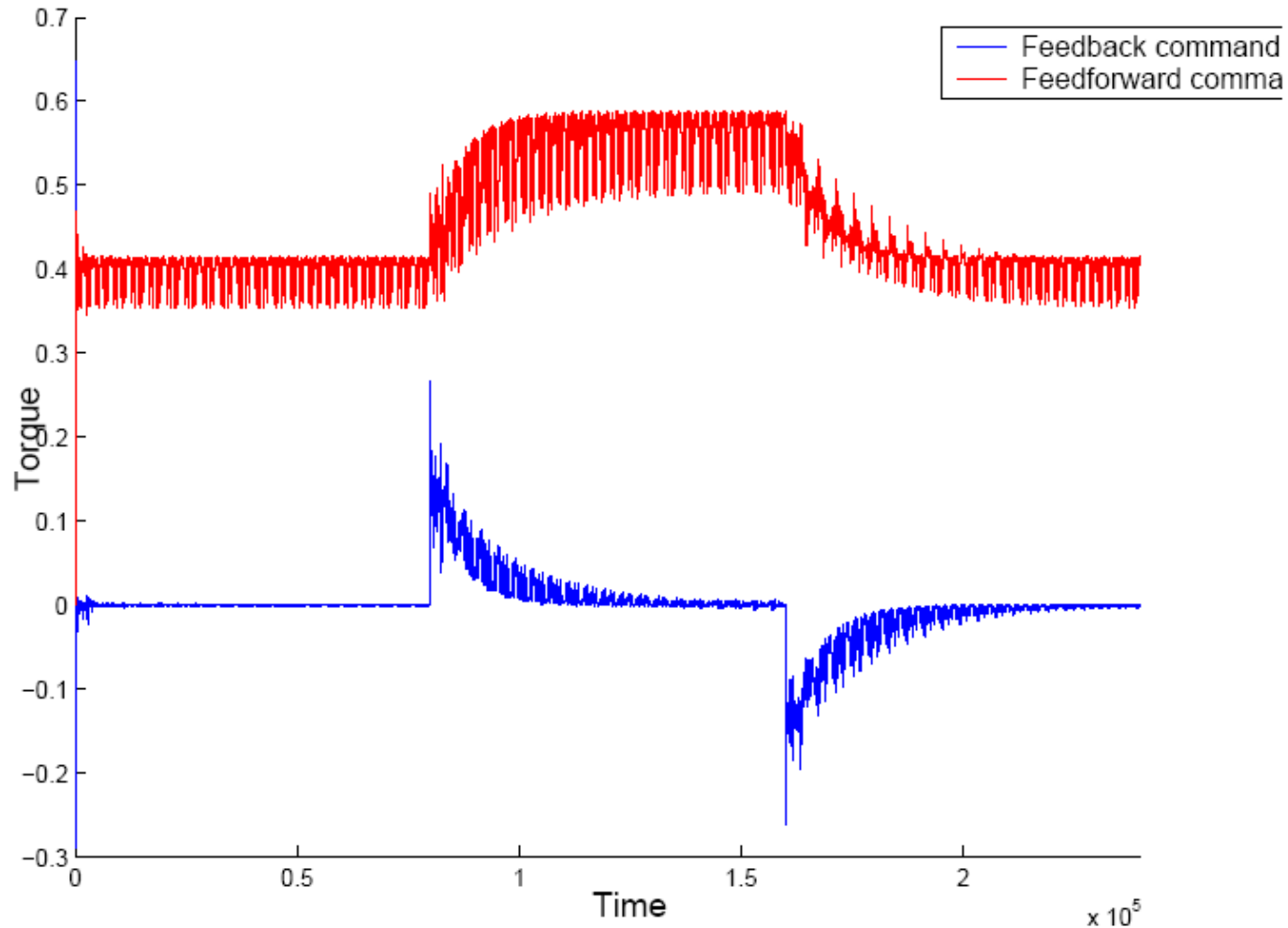
$$\tau_t = g(\theta_t^d, \theta_{t+1}^d) + K_p e_t + K_d \frac{de}{dt}$$

# Example of learning dynamics for control

- Simple 3 Dof arm
- Sinusoidal movement task
- Switch from PD to composite control at iteration 4
- Results averaged over 10 trials



# Recurring contexts?



- Tracking error behaves similarly

# Multiple Models



Learn one model for each different context and switch between them appropriately

Studies claim that the CNS uses multiple models and switches between them.

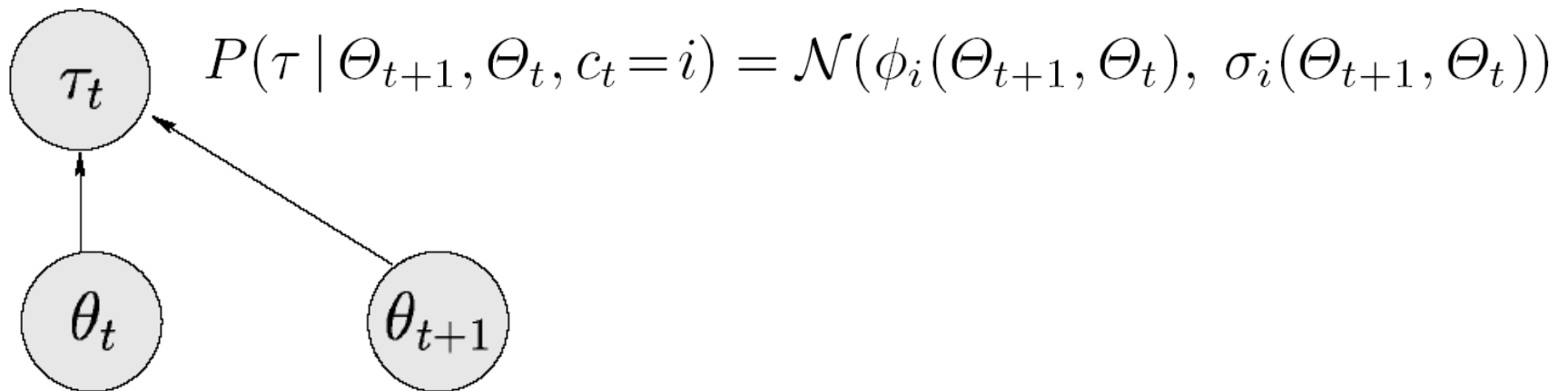
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# Existing multiple models approaches

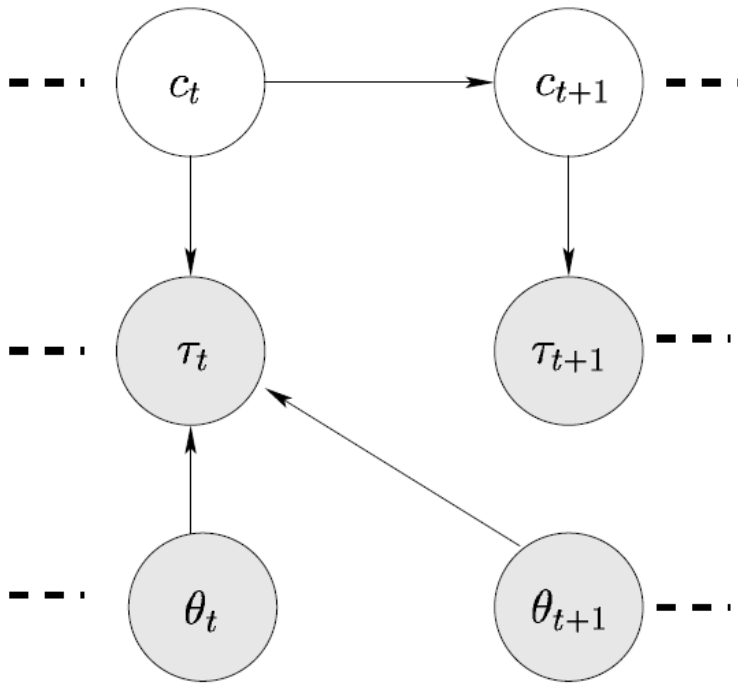
- MMST (Narendra) : limited learning, prior knowledge on possible dynamics required
  - MPFIM (Wolpert) : pairs of forward and inverse models, only shown to work with linear dynamics models, learning with online gradient descent
  - MOSAIC (Haruno) : similar to MPFIM, HMM used to smooth context estimates, trained with EM algorithm, shown to work with linear dynamics
-

# Multiple model formulation

$$P(\tau | \Theta_{t+1}, \Theta_t) = \mathcal{N}(\phi(\Theta_{t+1}, \Theta_t), \sigma(\Theta_{t+1}, \Theta_t))$$



# Incorporating temporal context

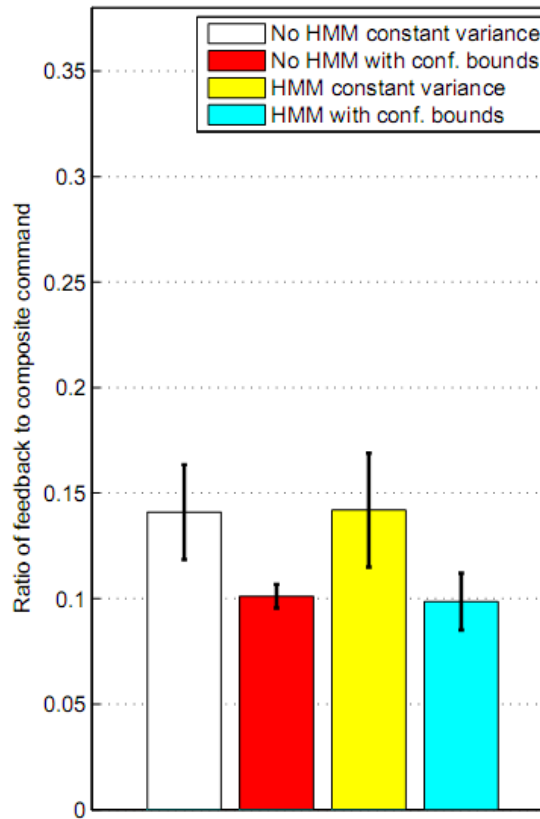
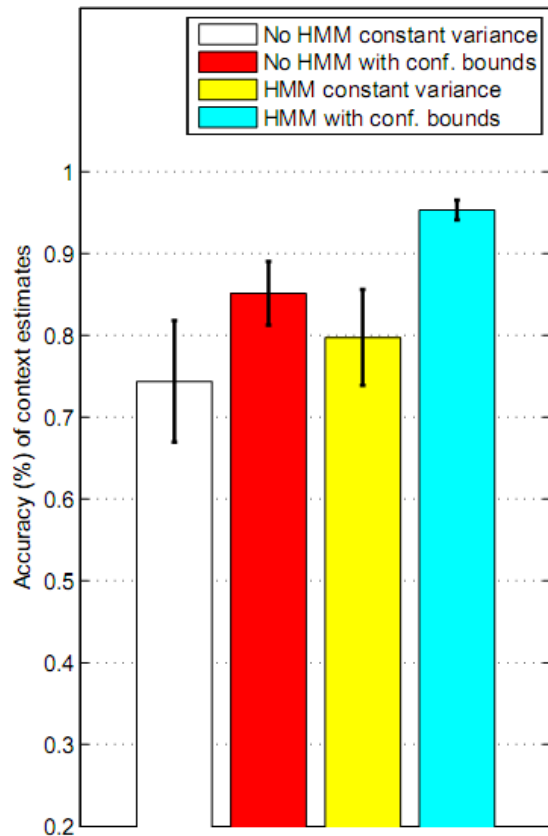




# Inference and learning

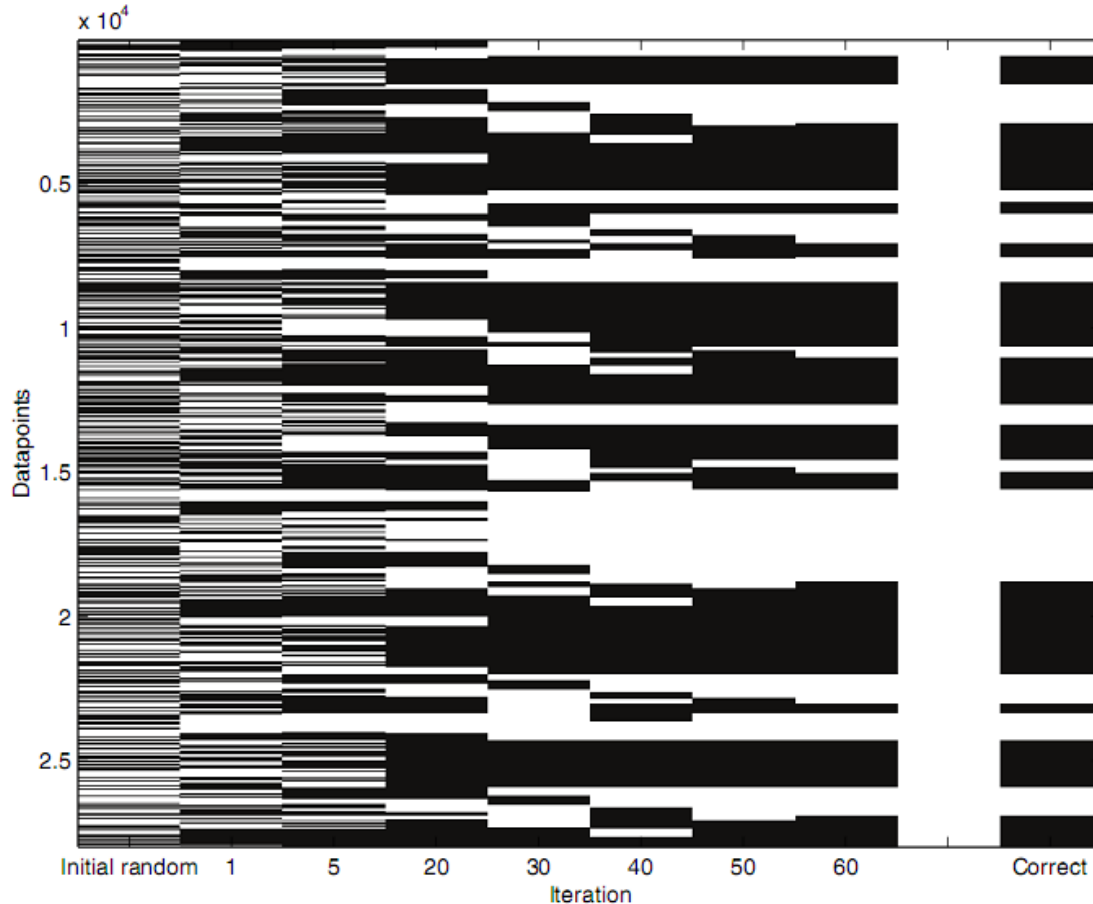
- Inference simply involves using Bayes rule / standard graphical model mechanisms (Viterbi alignment / filtering / smoothing)
  - Learning can be performed using EM (needs to be slightly modified when using LWPR)
-

# Experiments: context estimation



- Random switching between 10 contexts.
- Results averaged over 5 trials

# Experiments: learning



- Random switching between 2 contexts.
- Executed EM.
- Use of local models poses some difficulties.
- Need to switch from smoothed estimates to filtered estimates in the E-step.
- Confidence bounds used.

# Other multiple model approaches

## Previous methods

- Have **limited or no learning ability** (Multiple Model Switching and Tuning, Narendra)
- Have not been shown to work with **non-linear dynamics** (Modular Selection and Identification for Control, Wolpert and Kawato)

## Our approach

- Is able to learn nonlinear dynamics and **uses LWPR**, a robust local based nonlinear regression algorithm
- **Modified EM algorithm** in order to deal with the problems induced by using a local regression algorithm

# Issues with the multiple model scenario

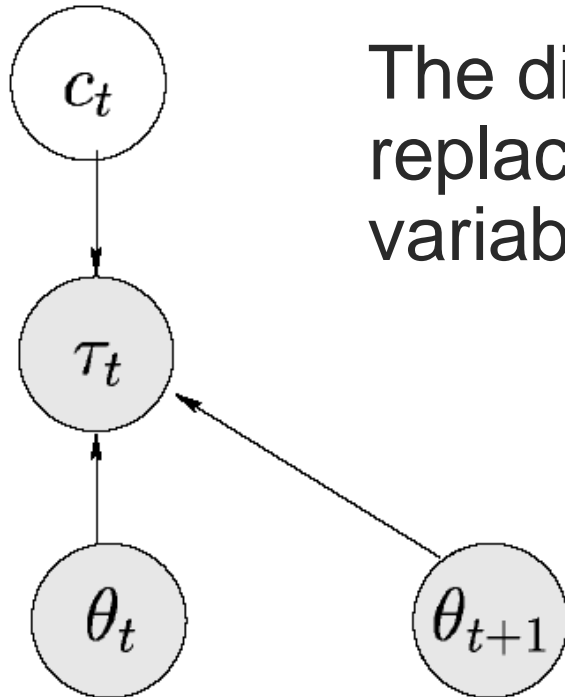
- How can we generalize to new contexts?
- What is the appropriate number of contexts?
- What if there is an infinite number of possible contexts?

Instead of a set of models use a single one with additional contextual information as input

$$\tau_t = G(\theta_t, \theta_{t+1}, c_t)$$

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# Reformulating the model



The discrete contextual variable is replaced by a set of continuous variables

$$\tau_t = G(\theta_t, \theta_{t+1}, c_t)$$

$$P(\tau | \theta_{t+1}, \theta_t, c_t = k) = \mathcal{N}(g_k(\theta_{t+1}, \theta_t), \sigma_k(\theta_{t+1}, \theta_t) c_t)$$

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# Learning the augmented model

- Use EM like before possible but too difficult
  - How many variables?
  - When possible it is better to use prior knowledge on the relationship of proper contextual variables to the dynamics
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# Special case: object manipulation

- This is possible in the case of manipulation of different objects (linear relationship of dynamics to a properly defined set of inertial parameters)

$$\sum_{j=1}^J y_{ij}(q, \dot{q}, \ddot{q})^T \pi_j = \tau_i$$

- $Y_{ij}$  are nonlinear functions of the joint angles, velocities and accelerations.
  - The vector  $\pi_j$  holds the inertial parameters of the  $j^{\text{th}}$  link (mass, positions of center of mass and inertia tensor)
-



# Special case: object manipulation

- Let us consider the dynamics of the context  $m$ .

$$\sum_{j=1}^J y_{ij}^m(q, \dot{q}, \ddot{q})^T \pi_j^m = \tau_i^m$$

- The  $y_{ij}$  s do not depend on the context, only on kinematic properties that do not change:

$$y_{ij}(q, \dot{q}, \ddot{q}) = y_{ij}^1(q, \dot{q}, \ddot{q}) = \dots = y_{ij}^M(q, \dot{q}, \ddot{q}), \forall i, j$$

- Only the inertial parameters of the last link change between contexts:

$$\pi_j = \pi_j^1 = \pi_j^2 = \dots = \pi_j^M, \text{ for } j = 1 \dots J - 1$$

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# Obtaining the augmented model

- Thus, the model for the  $m$ th context can be written as:

$$\left( \sum_{j=1}^{J-1} y_{ij}(q, \dot{q}, \ddot{q})^T \pi_j \right) + y_{iJ}(q, \dot{q}, \ddot{q})^T \pi_J^m = \tau_i^m$$



$$h_i(q, \dot{q}, \ddot{q}) + y_{iJ}(q, \dot{q}, \ddot{q})^T \pi_J^m = \tau_i^m$$

# Obtaining the augmented model

- Compiling the dynamics equations for all joints:

$$\begin{bmatrix} \tau_1^m \\ \tau_2^m \\ \vdots \\ \tau_{J-1}^m \end{bmatrix} = \begin{bmatrix} h_1(q, \dot{q}, \ddot{q}) \\ h_2(q, \dot{q}, \ddot{q}) \\ \vdots \\ h_J(q, \dot{q}, \ddot{q}) \end{bmatrix} + \begin{bmatrix} y_{1J}(q, \dot{q}, \ddot{q})^T \\ y_{2J}(q, \dot{q}, \ddot{q})^T \\ \vdots \\ y_{JJ}(q, \dot{q}, \ddot{q})^T \end{bmatrix} \pi_J^m$$

$$\tau^m = A(q, \dot{q}, \ddot{q}) + B(q, \dot{q}, \ddot{q}) \pi_J^m$$

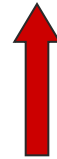
# Obtaining the augmented model

- Using  $\pi_j$  as the continuous context variables the augmented model can be:

$$\tau_t = G(\theta_t, \theta_{t+1}, c_t) = A(q, \dot{q}, \ddot{q}) + B(q, \dot{q}, \ddot{q})\pi_J^m$$

or

$$G(\theta_t, \theta_{t+1}, c_t) = \tilde{Y}(q, \dot{q}, \ddot{q})\tilde{\pi}_J^m = \tau$$



$$[A(q, \dot{q}, \ddot{q}) \quad B(q, \dot{q}, \ddot{q})]$$


$$[1 \quad \pi_J^m]$$

Obtaining the augmented model means obtaining the matrix  $\tilde{Y}$

# Obtaining the augmented model

- If we have  $M$  learned reference models  $\tau^1, \tau^2 \dots \tau^M$ , together with their corresponding context variables  $\pi^1, \pi^2 \dots \pi^M$  we can obtain  $Y$  as:

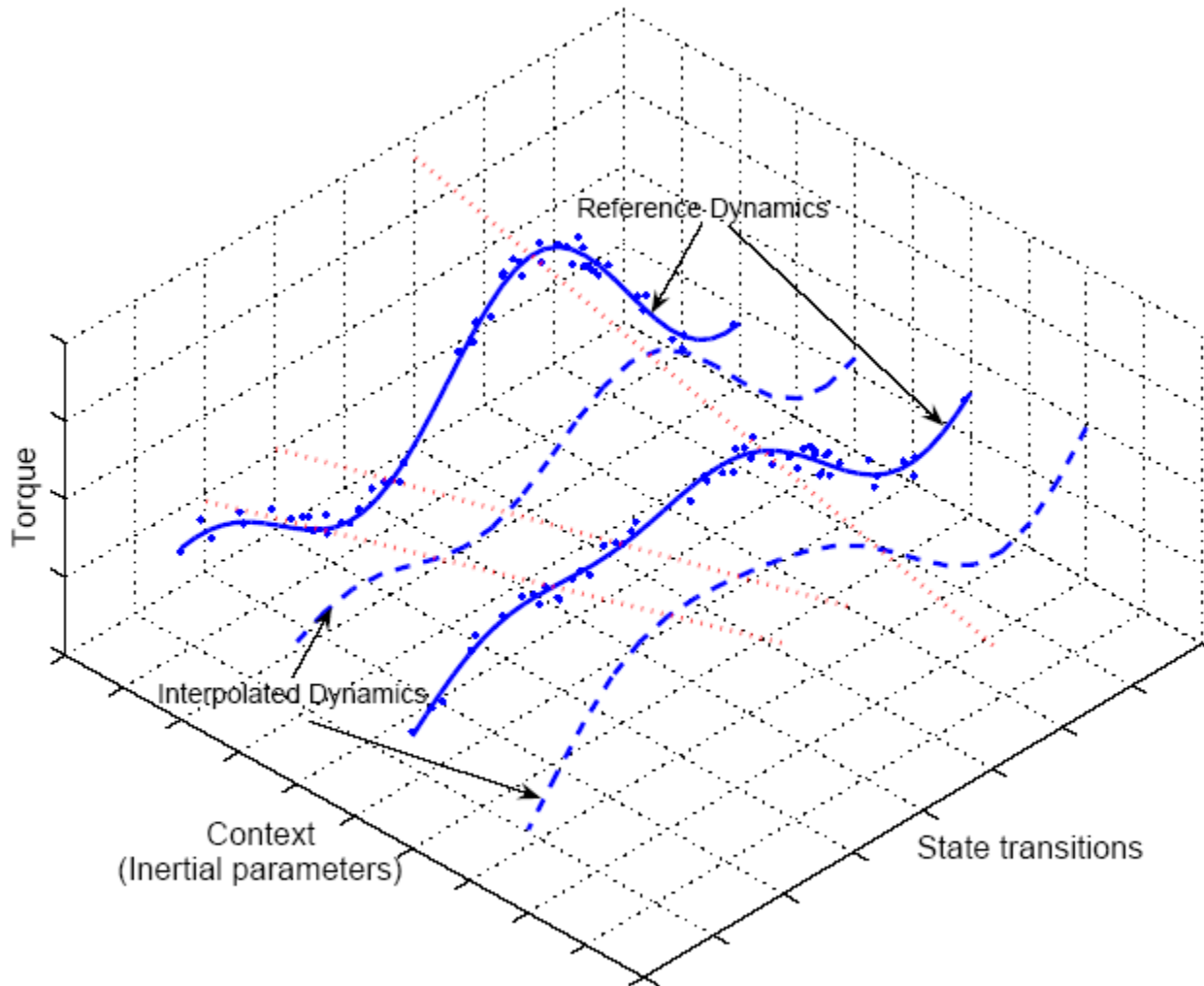
$$\begin{bmatrix} \tau^1 & \tau^2 & \dots & \tau^M \end{bmatrix} = \tilde{Y} \begin{bmatrix} \tilde{\pi}_J^1 & \tilde{\pi}_J^2 & \dots & \tilde{\pi}_J^M \end{bmatrix}$$


$$R = \tilde{Y}\tilde{\Pi}$$

$$\tilde{Y}(q, \dot{q}, \ddot{q}) = R\tilde{\Pi}^T (\tilde{\Pi}\tilde{\Pi}^T)^{-1} = R\tilde{\Pi}^+$$

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# Obtaining the augmented model



# Using the augmented model

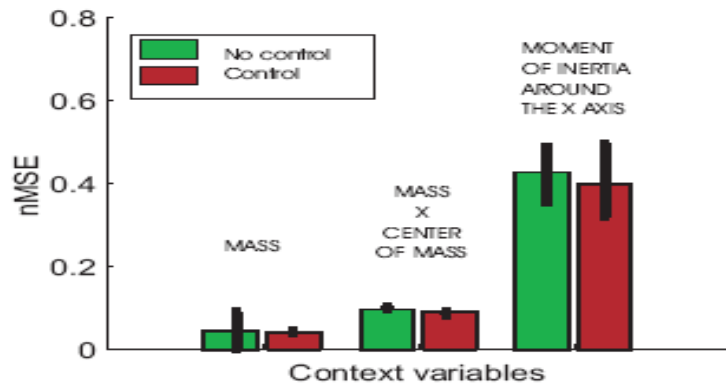
- Infer the current inertial parameters in a probabilistic manner similar to the discrete case.  
Can use:
    1. Non –temporal setting
    2. Temporal setting (similar to kalman filtering)
  - Use the estimated inertial parameters to get the feedforward command
-

# Experiments: continuous context

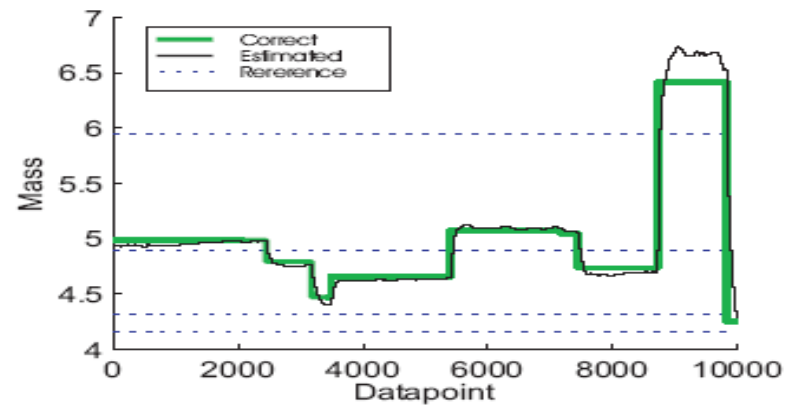
- Simulated 3 degree of freedom arm
  - Randomly varying mass and shape.
  - Inferred 3 inertial parameters
  - Used the estimates for control
  - 5 runs
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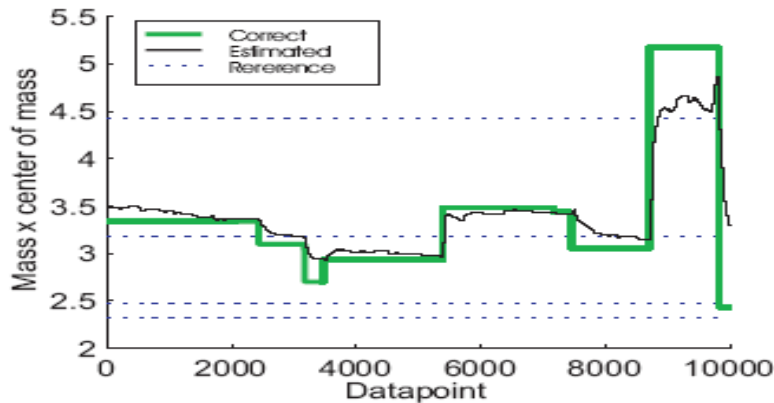
# Results



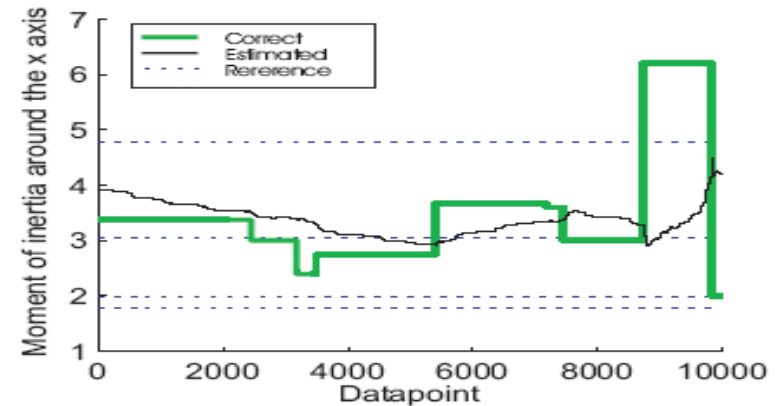
(a)



(b)



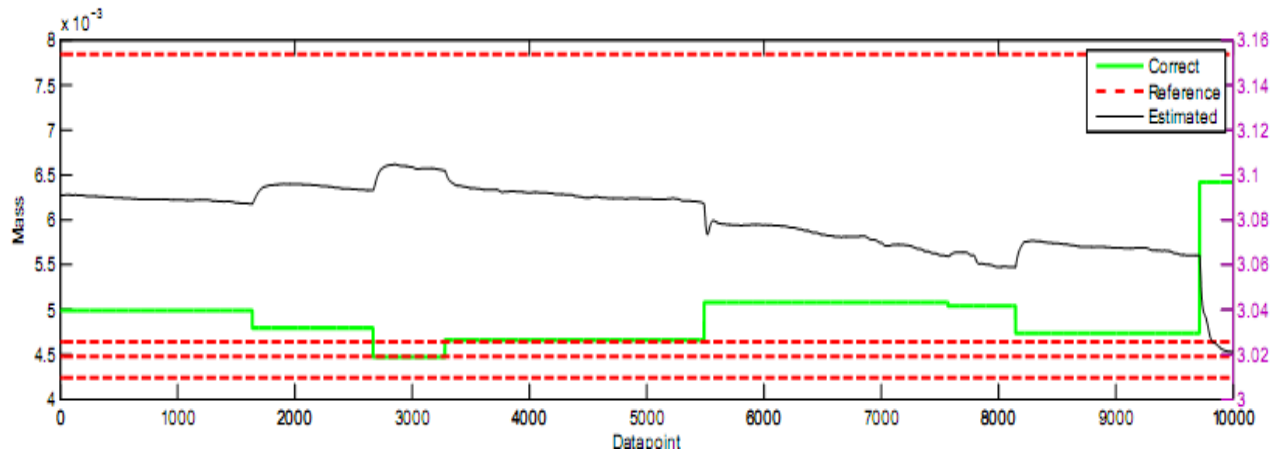
(c)



(d)

Ratio of feedback to composite command over the 3 joints was 0.1101 with std. 0.0176

# Unavailability of reference models' inertial parameters

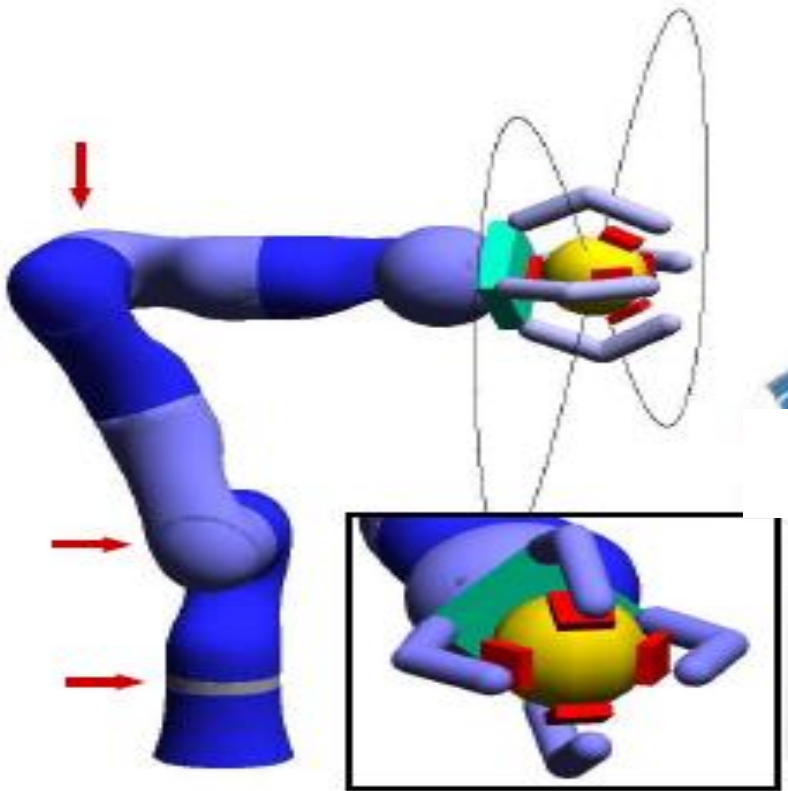


- Possible to reformulate so that reference models' inertial parameters are not required.
- Estimated context is a linear transformation of the actual context.
- Ratio of feedback to composite command 0.0893, std. 0.0239

# Remarks

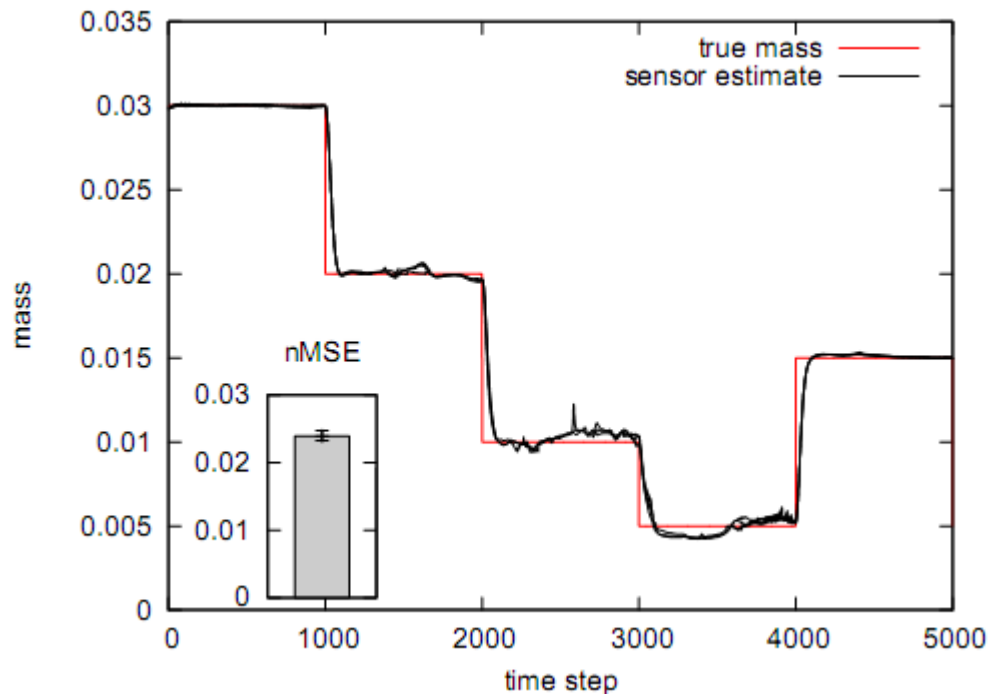
- This method is related to **classical load estimation** work on robotics
  - Some inertial parameters cannot be estimated
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# Using tactile sensing to estimate the context



- Simplistic model of tactile sensing is **linear in the mass of the manipulated object**
  - Use this prior knowledge to **generalize** across context estimates
  - Use context estimates with an augmented dynamics model for control

# Using tactile sensing to estimate the context

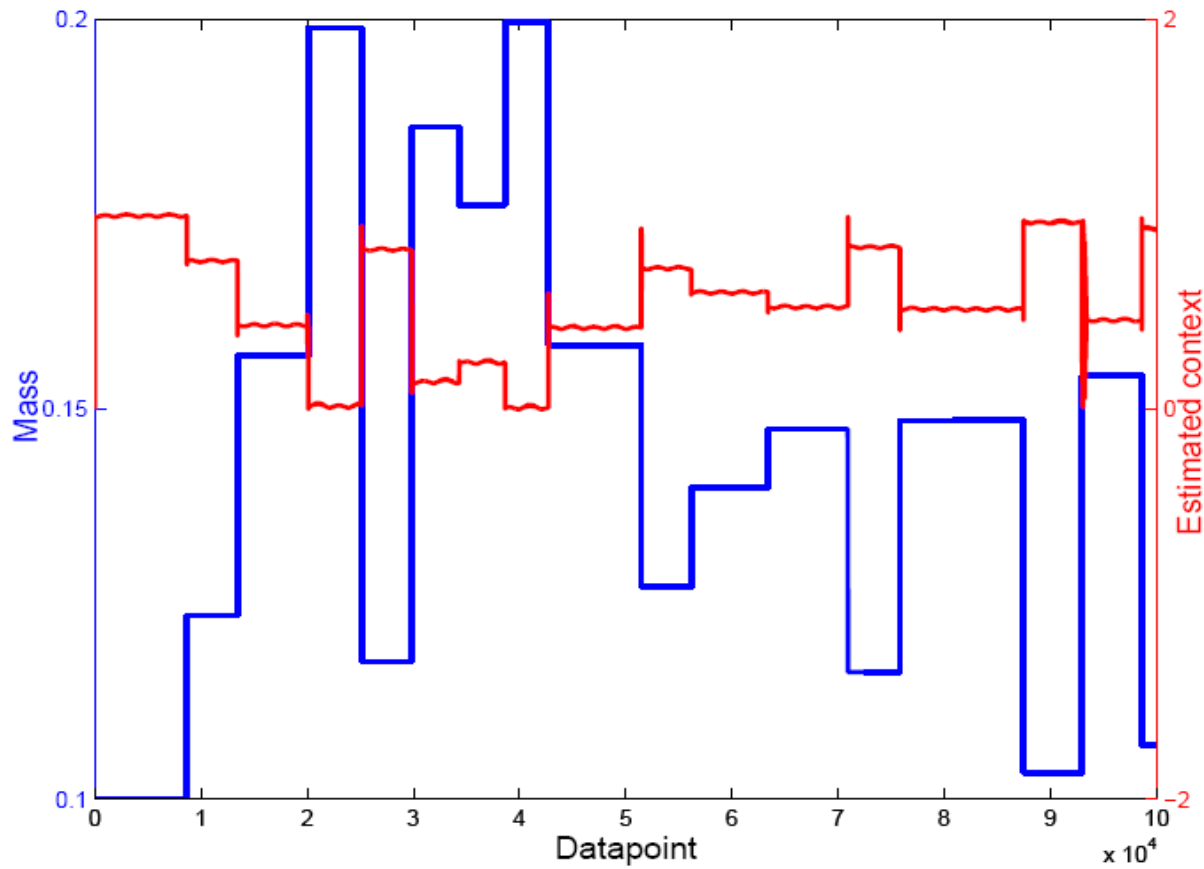


- Simulation of DLR arm
- Both sensor and augmented inverse models learned
- Three trials
- nMSE of estimates around 0.02

# Unknown nature of context

- Relationship of context to dynamics may be unknown, possibly nonlinear
  - Using the same temporal graphical model and just using a nonlinear augmented inverse model, we obtain a nonlinear state space model
  - Exact inference and maximum likelihood learning not possible
  - Possible choices for inference and learning include Extended Kalman Filtering, variational and Monte Carlo methods
  - We use a variant of Monte Carlo EM (only best particle used for training).
-

# Context estimation using a nonlinear state space model



- Simulation of simple 1DOF arm
- 100 particles
- 50 EM iterations
- Had to fix transition model in order to get a nice representation of the latent variable (mixture of uniform and gaussian with low noise variance).

# An alternative

- Previous approaches require to perform context estimation before applying a control command
- Could use an augmented model with observed variables that convey information about the context indirectly
- Tactile for varying loads

$$\tau = G(q, \dot{q}, \ddot{q}, T)$$

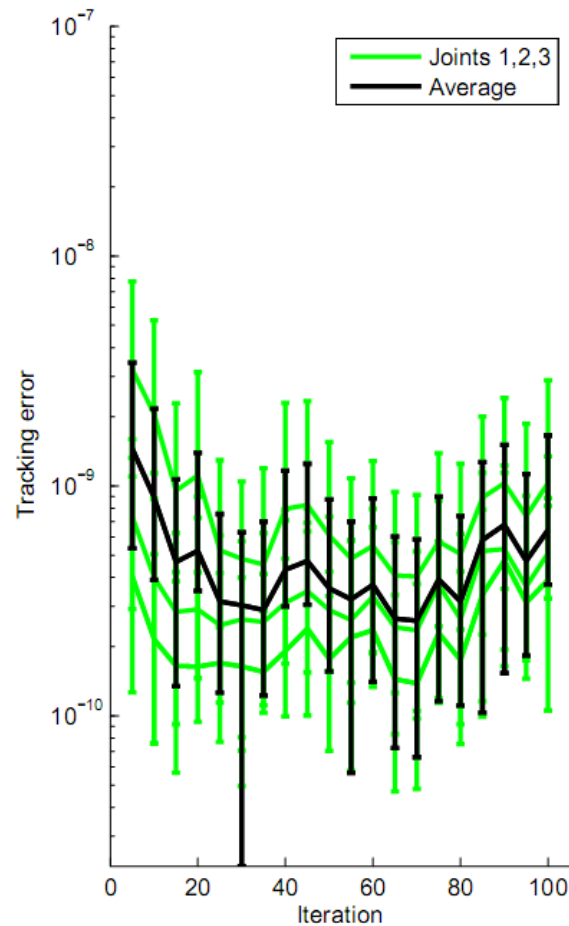
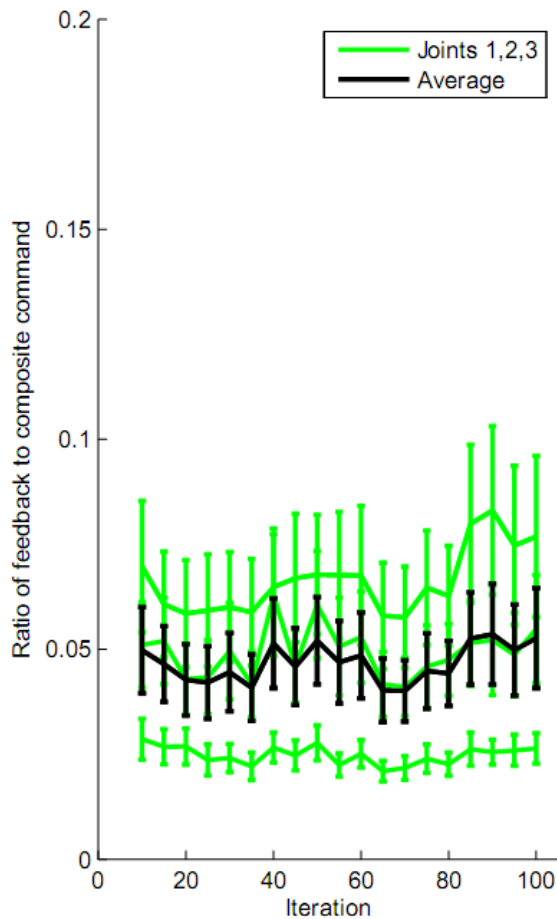
- Time delayed state transitions and commands. Essentially an autoregressive model

$$\tau_t = G(q_t, \dot{q}_t, \ddot{q}_t, q_{t-1}, \dot{q}_{t-1}, \ddot{q}_{t-1}, \tau_{t-1})$$

- Higher dimensional problem but context estimation is not required anymore
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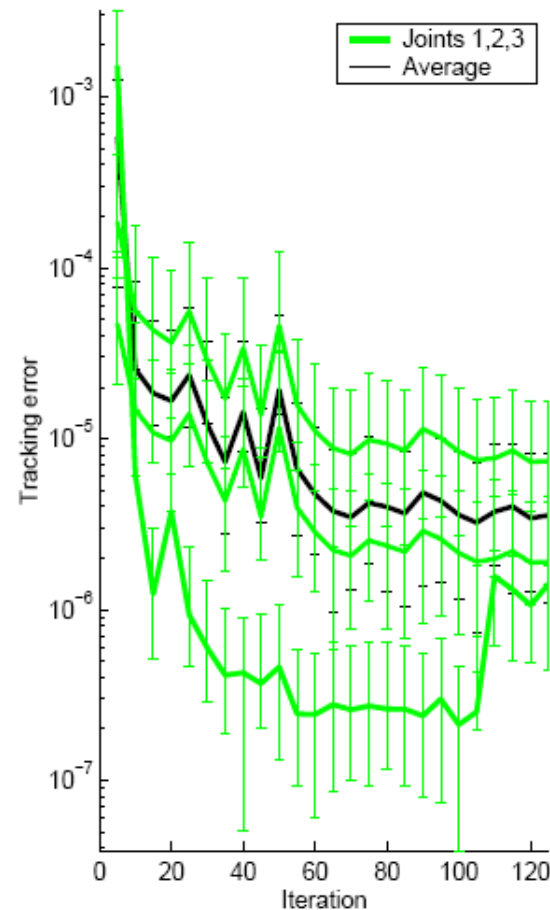
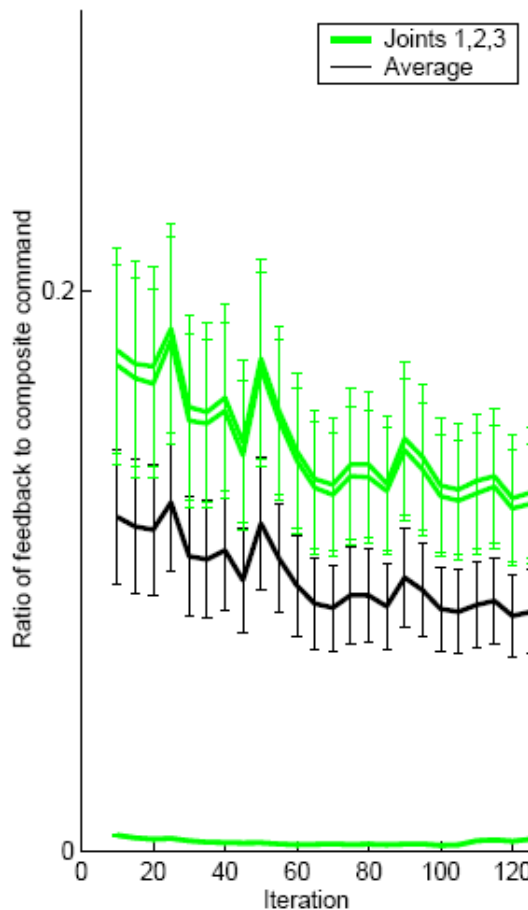


# Results using the tactile augmented model



- DLR arm
- Training stops at iteration 80

# Results using the autoregressive model

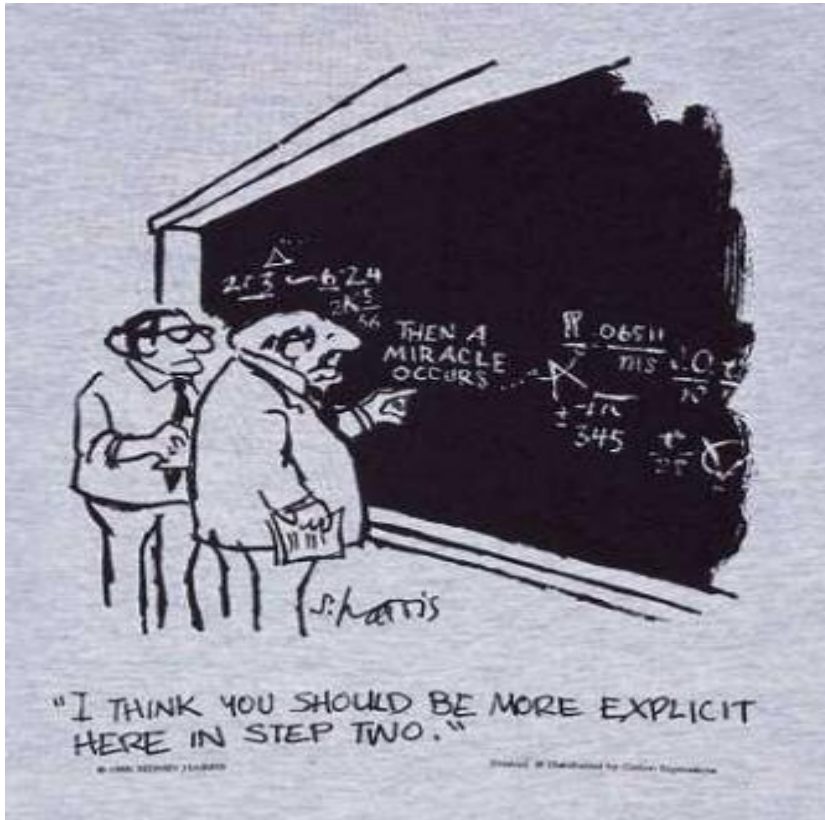


- Artificial arm 3DoF arm
- Training stops at iteration 100

# Summary

- **Learning of single model** for stationary and non-stationary dynamics
  - **Multiple models** for a set of recurring dynamics
  - **Continuous latent variable** model for a range of contexts
    - Linear state space model for a scenario when the nature of the context is known
    - Nonlinear state space model when the nature of the context is not known
  - Augmented model with variables that convey **contextual information indirectly** and are **observed**
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# Thank you!



- Questions?
- Comments?